

Additional Practice

Investigation 1

Variables and Patterns

1. a. The graph below shows the relationship between two variables. What are the variables?



- b. On which day were the most cans of food collected? About how many cans were collected on that day?
- c. What total number of cans was collected over the 5 days? Explain your reasoning.
- d. On this graph, does it make sense to connect the points with line segments? Explain your reasoning.

Additional Practice *(continued)***Investigation 1****Variables and Patterns**

2. Emma and her mother walk along a straight road from their house to their favorite ice cream shop. Emma keeps track of their pace over their seventy-five-minute walk. She made the following notes:
- We walked $\frac{3}{4}$ mile in the first 20 minutes.
 - We stopped for 10 minutes to talk to a friend.
 - For the next 20 minutes we walked more slowly and passed the $\frac{1}{2}$ mile of lovely gardens.
 - We walked at our normal pace for the next $\frac{3}{4}$ mile. This took 20 minutes.
 - We walked very fast for the last $\frac{1}{2}$ mile to get to the shop before it closed. This took only 5 minutes.
- a. Make a table of (*time, distance*) data that reasonably fits the information in Emma's notes.
- b. Sketch a coordinate graph that shows the same information as the table.
- c. Does it make sense to connect the points on this graph? Explain your reasoning.
- d. If Emma decided to only show one method of displaying the data (*time, distance*) to her mother, which should she choose if she wanted to show her mother the changes in their walking speed? Explain your choice.

Additional Practice *(continued)***Investigation 1****Variables and Patterns**

3. a. Andrew's mother kept the chart below of the number of words his sister Sarah could say at the end of each month from age 1 month to 24 months. Sarah did not say a word until 12 months, so from 1 to 11 Andrew's mother wrote 0. Make a coordinate graph of these data. Explain how you chose the variables for each axis.

Age (months)	Number of Words Sarah can Say
1–11	0
12	1
13	1
14	2
15	3
16	7
17	10
18	15
19	24
20	28
21	30
22	47
23	51
24	62

- b. Describe how the number of words Sarah can say changed as she got older (as the number of months passed).
- c. During what month did Sarah learn to say the most words? The least (not counting from 1 to 11 months)?

Additional Practice *(continued)***Investigation 1****Variables and Patterns**

4. The Student Council of Metropolis Middle School voted on seven ideas related to school activities. There are nine students on the Student Council and each student voted “yes” or “no” for each idea. Use the information in the table at the right to answer parts (a)–(d).

School Activity Ideas

Idea	Yes Votes
1	6
2	9
3	3
4	8
5	6
6	5
7	7

- a. What are the variables shown in the table?
- b. Which variable could be the independent variable and which could be the dependent variable? Explain your reasoning.
- c. Make a coordinate graph of the data in the table. Label your x -axis and y -axis with the correct independent or dependent variable.
- d. Make a coordinate graph showing how many students voted “no” on each of the seven ideas. Explain how you find the data for your graph. Label the x -axis and y -axis with the appropriate independent or dependent variable.

Additional Practice *(continued)*

Investigation 1

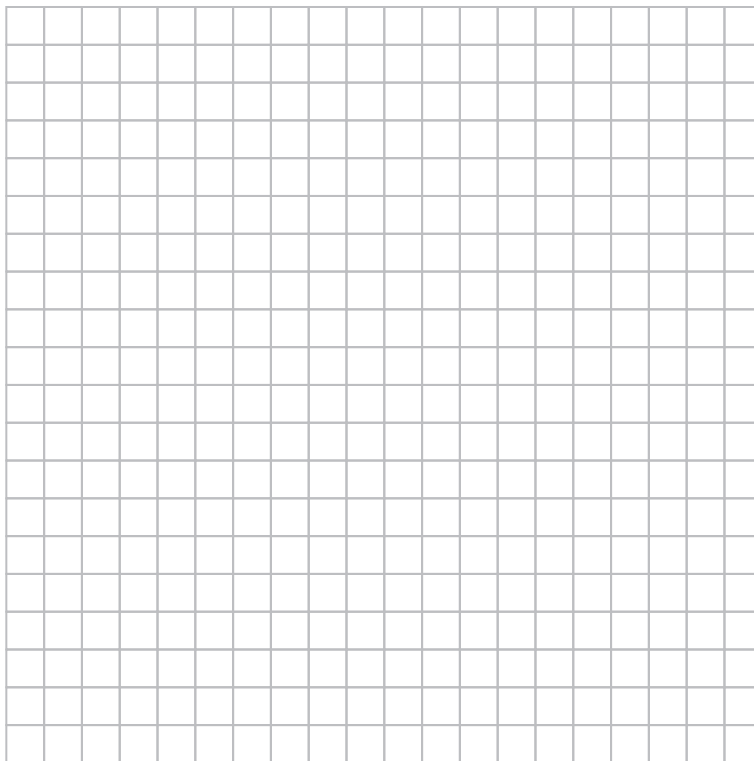
Variables and Patterns

5. Below is a chart of the water depth in a harbor during a typical 24-hour day.
The water level rises and falls with the tide.

Hours Since Midnight	0	1	2	3	4	5	6	7	8	9	10	11	12
Depth (meters)	8.4	8.9	9.9	10.7	11.2	12.1	12.9	12.2	11.3	10.6	9.4	8.3	8.0

Hours Since Midnight	13	14	15	16	17	18	19	20	21	22	23	24
Depth (meters)	8.4	9.4	10.8	11.4	12.2	13.0	12.4	11.3	10.4	9.8	8.6	8.1

- a. Make a coordinate graph of the data.



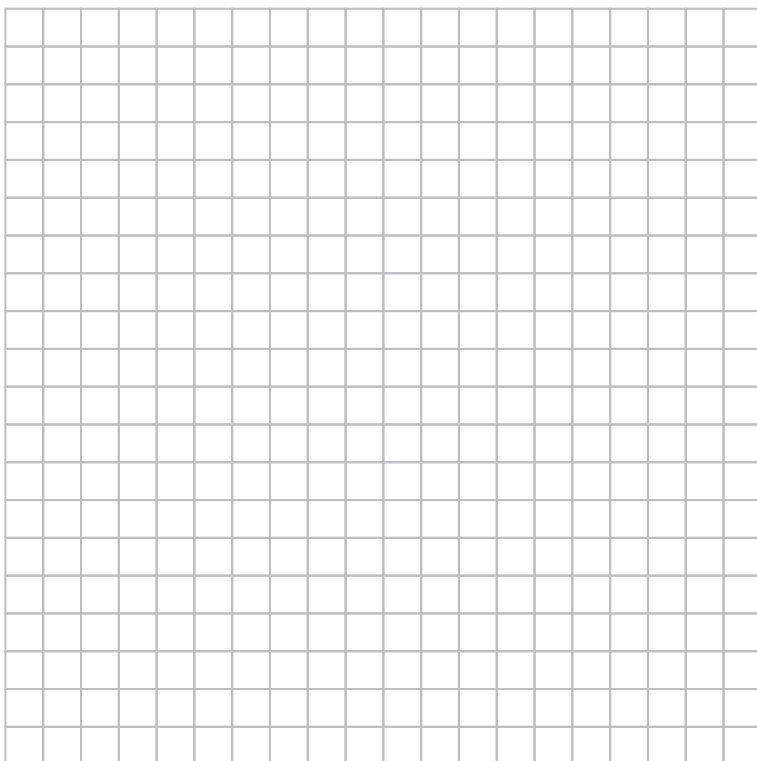
- b. During which time interval(s) does the depth of the water increase the most?
- c. During which time interval(s) does the depth of the water decrease the most?
- d. Would it make sense to connect the points on the graph? Why or why not?
- e. Is it easier to use the table or the graph to answer parts (b) and (c)? Explain.

Additional Practice *(continued)*

Investigation 1

Variables and Patterns

6. Make a table and a graph of (*time, temperature*) data that fit the following information about a day on the road:
- We started riding at 9:00 A.M. once the fog had burned off. The day was quite cool. The temperature was 52°F, and the sun was shining brightly.
 - About midmorning, the temperature rose to 70°F and cloud cover moved in, which kept the temperature steady until lunch time.
 - Suddenly the sun burst through the clouds, and the temperature began to climb. By late afternoon, it was 80°F.

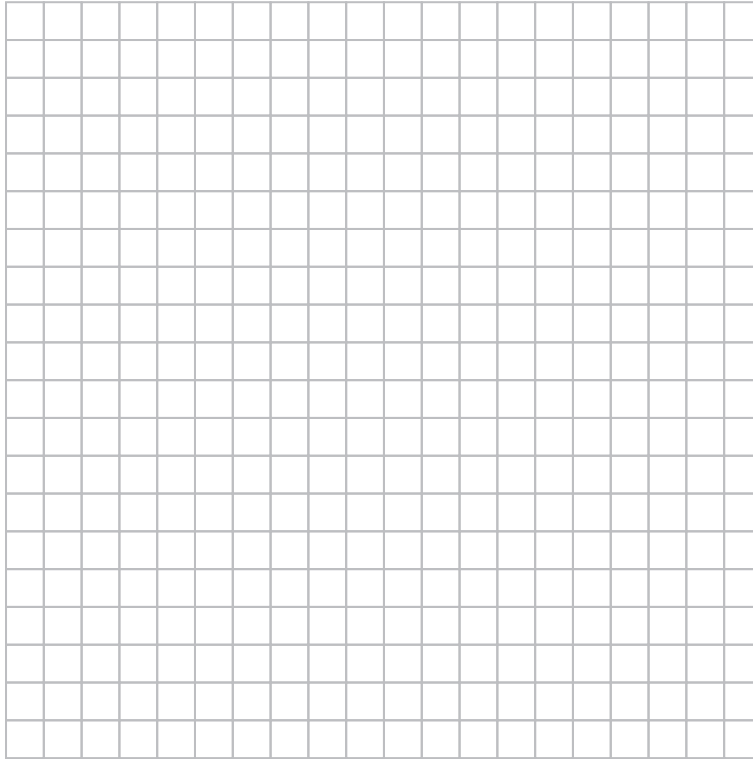


Additional Practice *(continued)*

Investigation 1

Variables and Patterns

7. Make a graph that shows your hunger level over the course of a day. Label the x -axis from 6 A.M. to midnight. Write a story about what happened during the day in relation to your hunger level.



Additional Practice *(continued)***Investigation 1****Variables and Patterns**

8. The following table shows (*time, distance*) data for a ten-hour drive that Mr. and Ms. Shah took from Cleveland, Ohio, to Nashville, Tennessee.

Time (hours)	0	1	2	3	4	5	6	7	8	9	10
Distance (miles)	0	70	125	195	252	275	280	343	413	465	525

- What was their average speed for the total trip?
- What was their average speed for the first five hours of the trip?
- What was their average speed for the second five hours of the trip?
- Sketch a coordinate graph that shows the same information as the table.
- What hour(s) of the trip did the Shahs travel the shortest distance?
Travel the farthest?
- Suppose the average speed of the final hour had been the average speed throughout the entire trip. Then how long would the trip have taken them?

Additional Practice: Digital Assessments

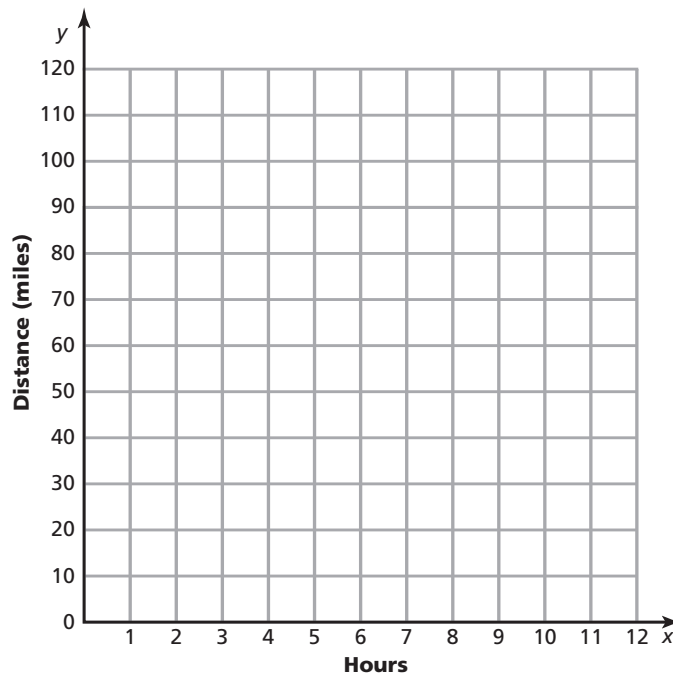
Investigation 1

Variables and Patterns

9. Carl rides his bike in a 12-hour cross-country race. The chart shows the total distance he rides by each hour mark.

Hours	0	1	2	3	4	5	6	7	8	9	10	11	12
Distance (miles)	0	14	26	35	47	51	57	64	77	85	94	101	116

a. Plot points on the coordinate grid to show the data from the chart.



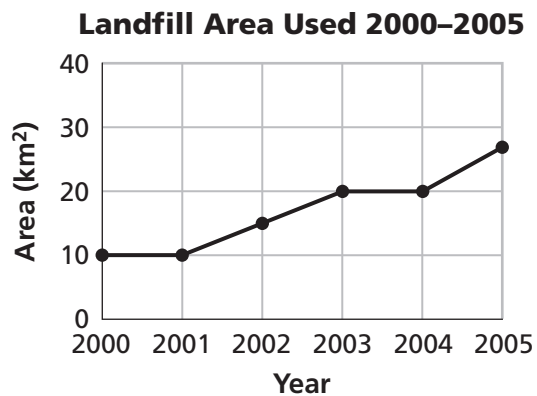
Circle the intervals that make each statement true.

b. Carl's fastest average speed is between hours $\left[\begin{array}{l} 0 \text{ and } 1 \\ 3 \text{ and } 4 \\ 4 \text{ and } 5 \\ 10 \text{ and } 11 \\ 11 \text{ and } 12 \end{array} \right]$.

c. Carl's slowest average speed is between hours $\left[\begin{array}{l} 0 \text{ and } 1 \\ 4 \text{ and } 5 \\ 5 \text{ and } 6 \\ 10 \text{ and } 11 \\ 11 \text{ and } 12 \end{array} \right]$.

Additional Practice *(continued)***Investigation 2****Variables and Patterns**

2. a. A newspaper included the graph below in a story about the amount of city land used for trash between 2000 and 2005. The graph shows the relationship between two variables. What are they?



- b. What is the difference between the least and greatest amount of land used for trash?
- c. Between which two years did the area used for trash stay the same?
- d. On this graph, what information is given by the lines connecting the points? Is this information necessarily accurate? Explain your reasoning.
- e. In 2000, the total area available for trash was 120 square kilometers. Make a coordinate graph that shows the landfill area remaining in each year from 2000 to 2005.

Additional Practice *(continued)*

Investigation 2

Variables and Patterns

3. a. Make a coordinate graph of these data.

Roller Rink Fees

Minutes	Cost
30	\$3.50
60	\$7.00
90	\$10.50
120	\$14.00
150	\$17.50
180	\$21.00



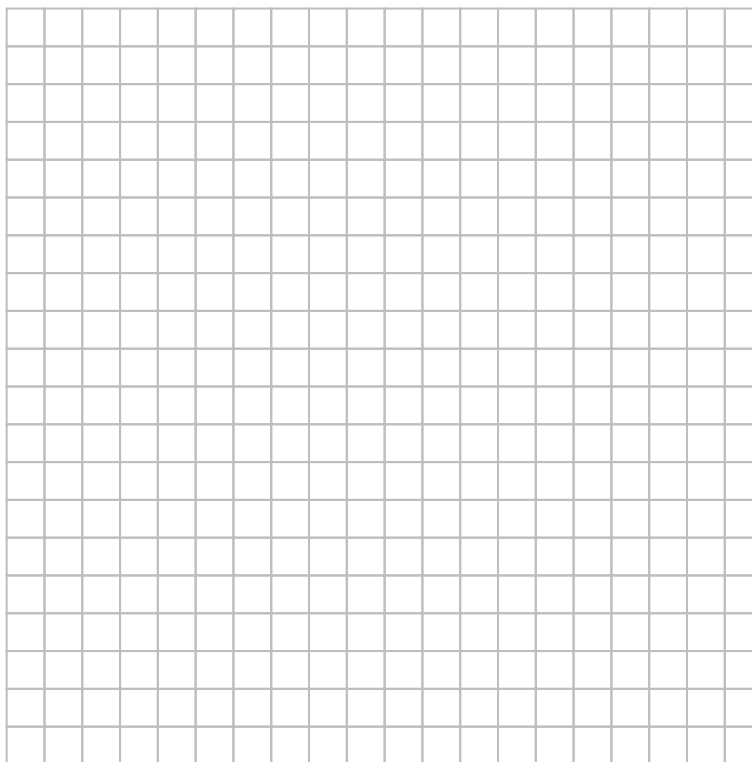
- b. Would it make sense to connect the points on your graph? Why or why not?
- c. Using the table, describe the pattern of change in the total skating fee as the number of minutes increases. How is this pattern shown in the graph?

Additional Practice *(continued)*

Investigation 2

Variables and Patterns

4. a. A roller-blade supply store rents roller blades for \$2.50 per skater. Using increments of 5 skaters, make a table showing the total rental charge for 0 to 50 skaters. Make a coordinate graph of these data.



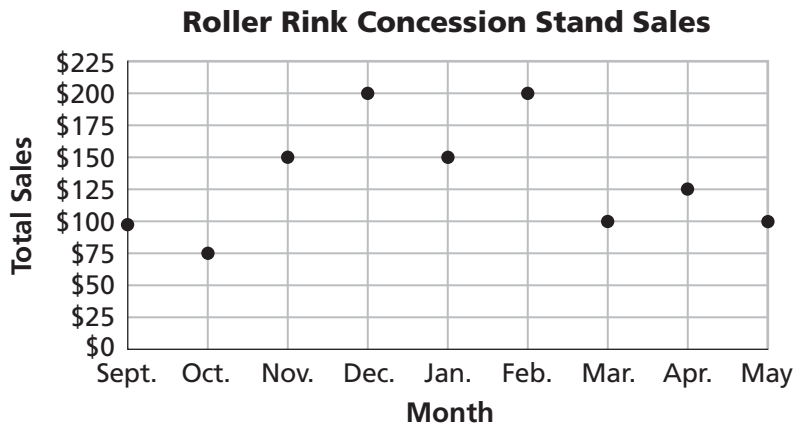
- b. Compare the pattern of change in your table and graph with the patterns you found in the skating fees in Exercise 3. Describe any similarities and differences.

Additional Practice *(continued)*

Investigation 2

Variables and Patterns

5. a. Use the graph to make a table of data showing the sales for each month.



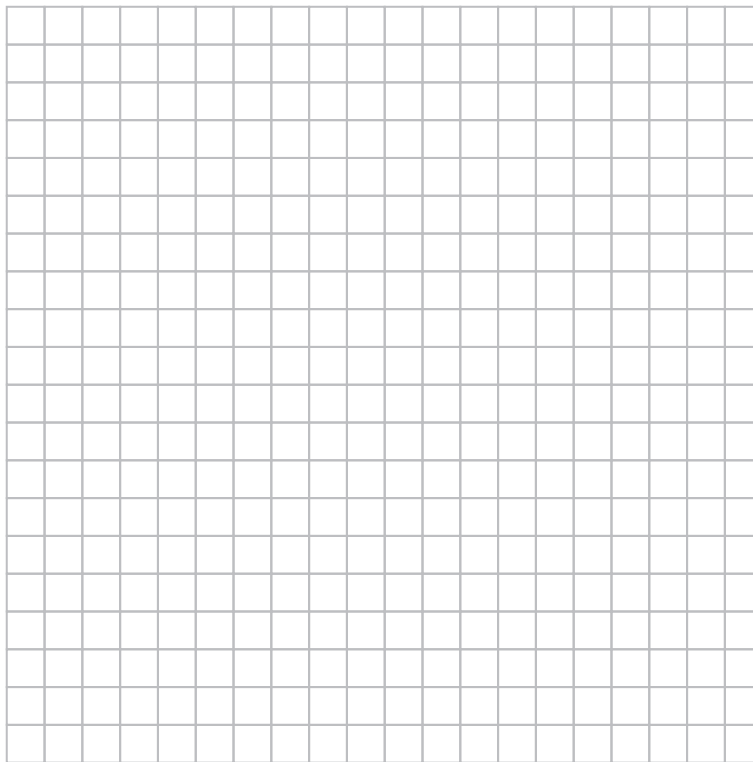
b. The profit made by the concession stand is half of the sales. Make a table of data that shows the profit made by the concession stand for each month.

Additional Practice *(continued)*

Investigation 2

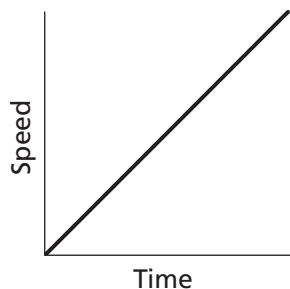
Variables and Patterns

- c. Make a coordinate graph of the data from part (b). Use the same scale used in the sales graph above. Describe how the sales graph and the profit graph are similar and how they are different.

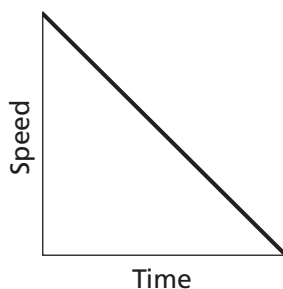


6. The three graphs below show the progress of a cyclist at different times during a ride. For each graph, describe the rider's progress over the time interval.

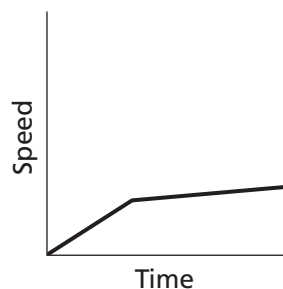
a.



b.



c.

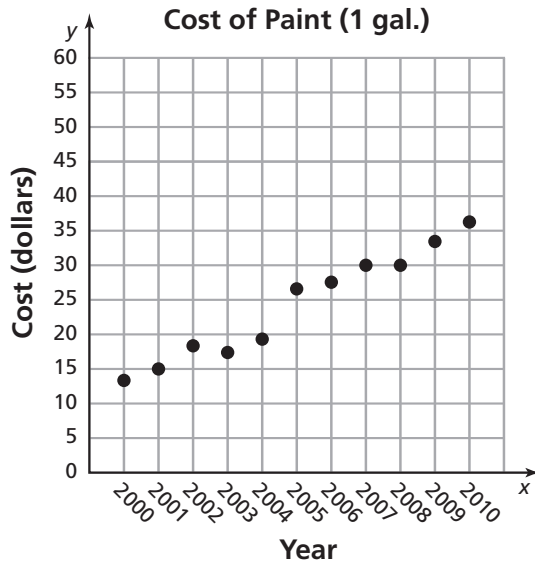


Additional Practice: Digital Assessments

Investigation 2

Variables and Patterns

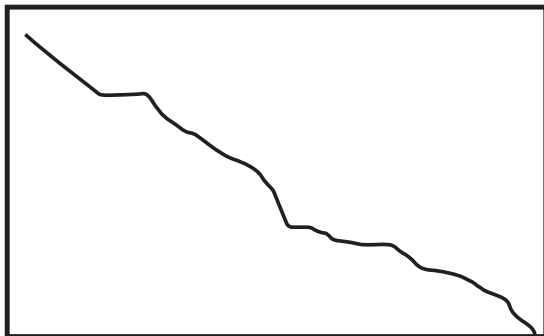
7. The graph below shows the relationship between two variables.
Use the words and numbers in the bank to complete each statement.



cost	gallon	paint	year
2000	2001	2002	2003
2004	2005	2006	2007
2008	2009	2010	

- a. The two variables shown in the graph are and .
- b. Between and there was no change in the cost of a gallon of paint.
- c. Between and the cost of a gallon of paint had the greatest increase.

8. Which of the following situations might be correctly modeled by the graph below?



Select all that apply.

- height of a feather as it falls from a flying bird
- total distance traveled over time
- total distance remaining to travel over time
- path of a basketball through a hoop
- snow accumulation during a blizzard

Skill: Tables and Graphs

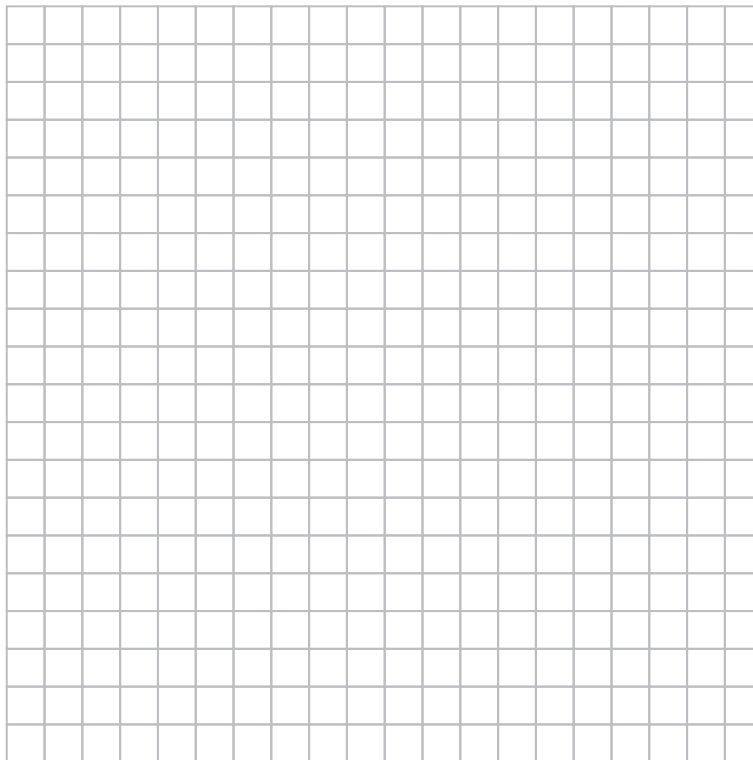
Investigation 2

Variables and Patterns

1. a. Graph the data in the table.

Storage Disks

Number of disks	Price (dollars)
1	20
2	37
3	50
6	100
10	150



b. Use the graph to estimate the cost of five disks.

Skill: Tables and Graphs *(continued)*

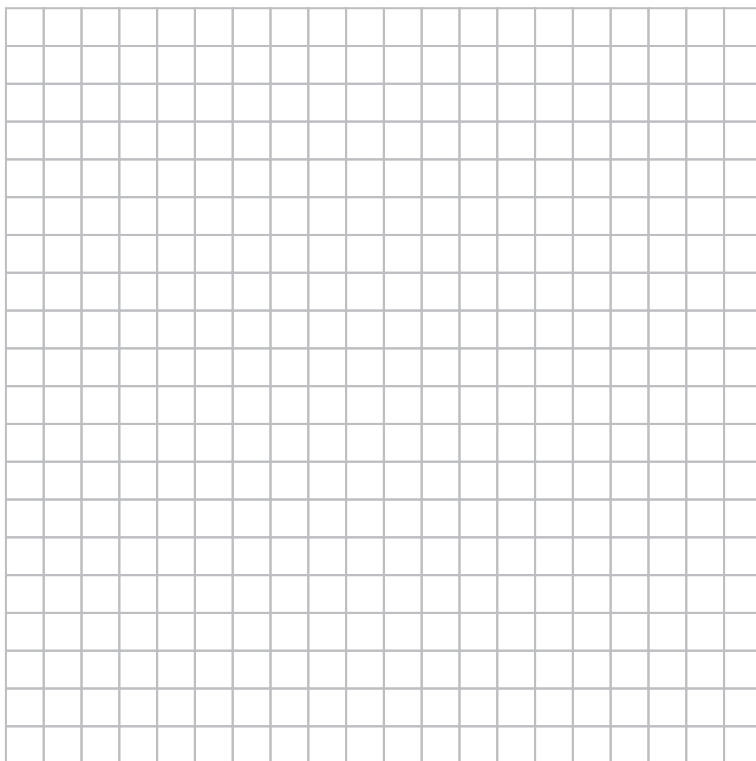
Investigation 2

Variables and Patterns

2. a. The table shows average monthly temperatures in degrees Fahrenheit for American cities in January and July. Graph the data in the table.

City	Seattle	Baltimore	Boise	Chicago	Dallas	Miami	LA
Jan.	39.1	32.7	29.9	21.4	44.0	67.1	56.0
Jul.	64.8	76.8	74.6	73.0	86.3	82.5	69.0

City	Anchorage	Honolulu	New York	Portland	New Orleans
Jan.	13.0	72.6	31.8	21.5	52.4
Jul.	58.1	80.1	76.4	68.1	82.1



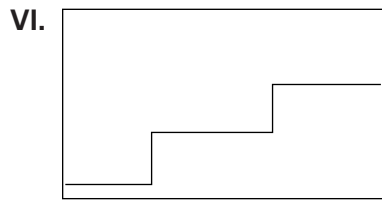
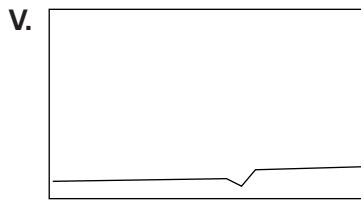
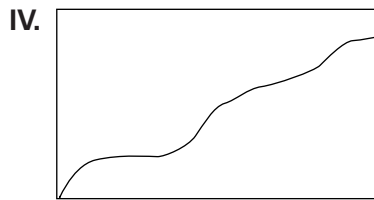
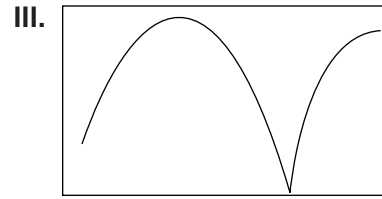
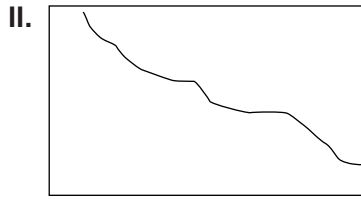
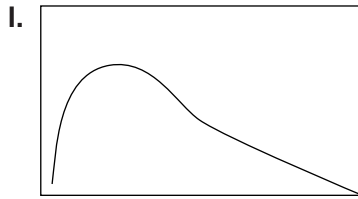
b. Use your graph to estimate the July temperature of a city whose average January temperature is 10°F.

Skill: Analyzing Graphs

Investigation 2

Variables and Patterns

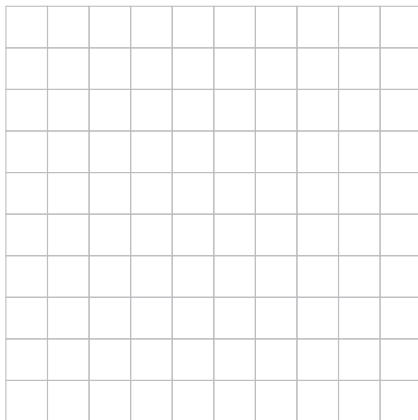
Graphs I through VI represent one of the six situations described below. Match each graph with the situation that describes it.



1. temperature as the weather changes from rainy to snowy
2. number of fish caught per hour on a bad fishing day
3. total rainfall during a rainy day
4. speed of a car starting from a stop sign and then approaching a stoplight
5. height of a cricket as it jumps
6. total amount of money spent over time during a trip to the mall

Sketch a graph for each situation.

7. The speed of a runner in a 1-mi race.



8. The height above ground of the air valve on a tire of a bicycle ridden on flat ground. (You can model this using a coin.)



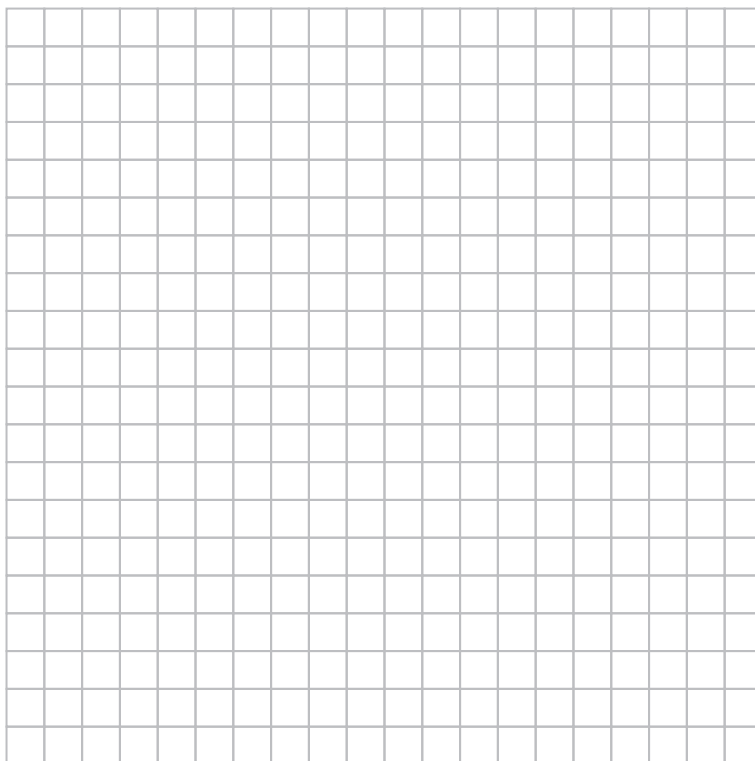
Additional Practice *(continued)*

Investigation 3

Variables and Patterns

2. The equation $d = 44t$ represents the distance in miles covered, after traveling 44 miles per hour for t hours.
- Make a table that shows the distance traveled, according to this equation, for every half hour between 0 and 4 hours.

- Sketch a graph that shows the distance traveled between 0 and 4 hours.



- If $t = 2.5$, what is d ?
- If $d = 66$, what is t ?
- Does it make sense to connect the points on this graph with line segments? Why or why not?

Additional Practice *(continued)*

Investigation 3

Variables and Patterns

3. a. The number of students at Smithville Middle School is 21 multiplied by the number of teachers. Use symbols to express the rule relating the number of students and the number of teachers as an equation. Use single letters for your variables and explain what each letter represents.
- b. If there are 50 teachers at Smithville Middle School, how many students attend the school?
- c. If 1,260 students attend Smithville Middle School, how many teachers teach at the school?

Additional Practice *(continued)***Investigation 3****Variables and Patterns**

4. a. Refer to the table below. Use symbols to express the rule relating the side length of a square to its area as an equation. Use single letters for your variables, and explain what each letter represents.

Squares

Side Length (cm)	Area (cm ²)
1	1
1.5	2.25
2	4
2.5	6.25
3	9

- b. Use your equation to find the area of a square with a side length of 6 centimeters.

- c. Use your equation to find the side length of a square with an area of 1.44 square centimeters.

Additional Practice: Digital Assessments

Investigation 3

Variables and Patterns

5. Which of the following input-output pairs follow the rule?

Rule: Output = Input • 3

Select all that apply.

- For an input of 6, the output is 18.
- Input = 3
Output = 1
- An input of 4 has an output of 12.
- When the input is 2, the output is 10.
- Input = 5
Output = 10
- Input = 1
Output = 3

6. Using the numbers on the tiles, complete the table.

2	8	12	15
20	30	32	40

x	$2.5x$
4	10
	20
12	
16	
	50

7. Match each situation to an equation.

Two batches of muffins use 5 cups of flour.

$$5x = 2y$$

The number of feet in a distance expressed in yards is three times the number of yards.

$$y = 5 + 2x$$

1 cup of juice concentrate is needed to make 5 quarts of punch.

$$y = 5x$$

The cost of going to the county fair is a \$5-entrance fee and \$2 per ride.

$$x = 3y$$

Skill: Variables, Tables, and Graphs**Investigation 3****Variables and Patterns**

In programming, an input can be thought of as the independent variable. An output can be thought of as the dependent variable. Complete each table given the rule.

1. Rule:
- $\text{Input} \cdot 5 = \text{Output}$

Input	1	2	3	4	5
Output	5	10	15		

2. Rule:
- $\text{Input} \cdot 2 = \text{Output}$

Input	10	20	30	40	50
Output	20	40	60		

3. Rule:
- $\text{Input} + 3 = \text{Output}$

Input	3	4	5	6	7
Output	6	7	8		

4. Rule:
- $\text{Input} + 1 = \text{Output}$

Input	6	7	8	9	10
Output	7	8	9		

5. Rule:
- $\text{Input} - 4 = \text{Output}$

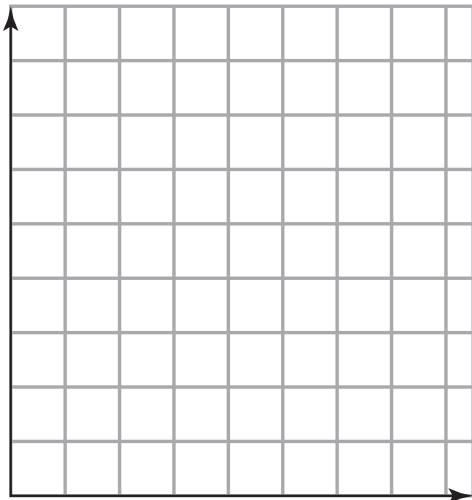
Input	12	13	14	15	16
Output	8	9	10		

Skill: Variables, Tables, and Graphs *(continued)*

Investigation 3

Variables and Patterns

6. A parking garage charges \$3.50 per hour to park. The equation $c = 3.5h$ shows how the number of hours h relates to the parking charge c . Graph this relationship.



Use the expression to complete each table.

7.

x	$x + 7$
2	9
5	12
8	
11	
	21

8.

x	$5x$
3	
6	
9	
12	
	75

9.

x	$125 - x$
15	
30	
45	
60	
	50

Skill: Variables, Tables, and Graphs *(continued)***Investigation 3****Variables and Patterns**

10. A cellular phone company charges a \$49.99 monthly fee for 600 free minutes. Each additional minute costs \$0.35. This month you used 750 minutes. How much do you owe?

Write a rule for the relationship between the variables represented in each table.

11.

x	y
1	6
2	7
3	8
4	9

12.

x	y
1	4
2	8
3	12
4	16

13.

x	y
1	4
2	7
3	10
4	13

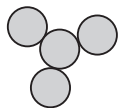
14. A typist types 45 words per minute.
- Write a rule to represent the relationship between the number of typed words and the time in which they are typed.
 - How many words can the typist type in 25 minutes? Write and solve an equation to answer this.
 - How long would it take the typist to type 20,025 words?

Additional Practice

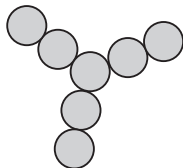
Investigation 4

Variables and Patterns

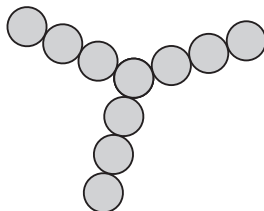
Use the shape pattern to complete Exercises 1 and 2.



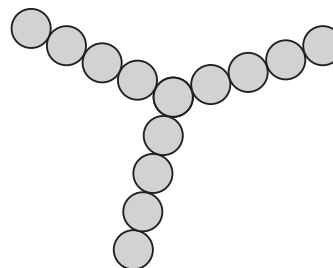
Shape 1



Shape 2



Shape 3



Shape 4

1. a. Complete the table comparing the shape number to the number of circles.

Shape Number	4	5	6	7	8	10	20
Number of Circles	13						

- b. Write an equation that could be used to find the number of circles C needed for shape number n . Explain how your equation relates to the shape pattern.
- c. How many circles are needed for shape number 40? Explain how you found your answer.
- d. What shape number requires 28 circles? Explain how you found your answer.

Additional Practice *(continued)***Investigation 4****Variables and Patterns**

2. a. Grace wrote the equation $C = 4n$ to represent the shape pattern. Is Grace's equation correct? Explain how Grace might have arrived at her equation.
- b. Write a different equation from Exercise 1(b) that represents the shape pattern.
- c. Show that the two equations are equivalent.

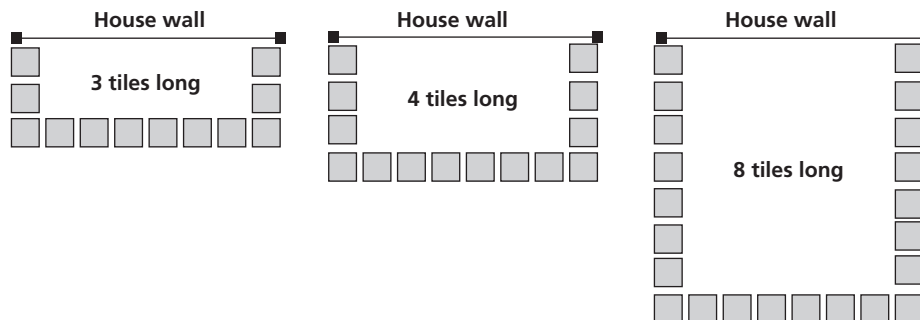
Additional Practice *(continued)*

Investigation 4

Variables and Patterns

Use the information to complete Exercises 3 and 4.

Brandon is constructing a garden using the side of the house as a wall. He isn't sure how long to make the garden. Here are sketches of three designs using 1-foot square tiles to surround the three sides of the garden.



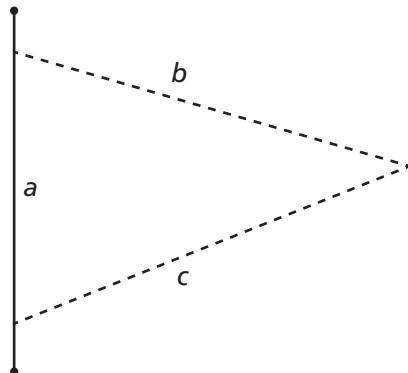
3. a. Complete the table comparing the length to the number of tiles needed to surround it.

Length of Garden in Tiles	3	4	5	6	7	8	10	20
Number of Tiles Needed	12	14						

- b. What is an equation you can use to find the number of tiles F needed for a garden that is n tiles long?
- c. Could you calculate the number of tiles needed for a garden that is 1 tile long? Explain.
4. Brandon and his friends wrote equations to represent the number of tiles needed for a garden that is n feet wide. Which equations represent the pattern?
 Brandon's equation: $F = 8 + n + n$
 Linda's equation: $F = 2n + 6$
 Alex's equation: $F = 8 + 2(n - 1)$

Additional Practice *(continued)***Investigation 4****Variables and Patterns**

8. Jamie has 12 inches of bendable wire. She wants to use the wire to form two sides of a triangular frame. Part of a 6-inch stick will form the third side with length a as shown.



- a. Write an equation that represents how to find the perimeter P of the triangle above.
- b. Write an equation that represents how sides b and c are related to the length of the bendable wire.
- c. If the two ends of the wire are glued to the two ends of the stick, what is the perimeter of the triangle? Write an equation to represent this perimeter. Can the perimeter be greater than this? Why or why not?
- d. Write an inequality to solve for possible values of a .

Additional Practice *(continued)***Investigation 4****Variables and Patterns**

9. A painter charges \$200 for materials and \$40 per hour for painting a house. He charged the Fishers \$400 for painting their house. Write and solve an equation to show how many hours the painter worked.

The library book fair charges a \$5 entry fee and \$1 per book. A school book fair charges \$2 per book. Use this information for Exercises 10–13.

10. For each book fair, write an equation to show how total cost C depends on the number b of books purchased.
11. Henry spent \$17 at the library book fair.
- Write and solve an equation to show how many books Henry purchased.
 - Explain how you found your solution.
 - Check your solution by substituting its value for b in the equation.

Additional Practice *(continued)*

Investigation 4

Variables and Patterns

12. Lucy spent \$12 at the school book fair.
- Write and solve an equation to show how many books Lucy purchased.
 - Explain how you found your solution.
 - Check your solution by substituting its value for b in the equation.
13. a. Parker has \$10 to spend. Write and solve an equation to show the most books he can purchase at the library book fair.
- Parker has \$10 to spend. Write and solve an equation to show the most books he can purchase at the school book fair.
 - Which book fair do you think Parker should attend? Explain your reasoning.

Additional Practice *(continued)***Investigation 4****Variables and Patterns**

Use what you know about variables, expressions, and equations to write and solve inequalities that match exercises 14–16. In each case, do the following:

- a. Write an inequality that helps to answer the question.
 - b. Give at least 3 specific number solutions to the inequality. Then explain why they work.
 - c. Describe all possible solutions.
14. Belinda is constructing a square garden. What are the possible side lengths of the garden if she can use up to 60 feet of fencing?
15. Geoff bought a shelf that will fit 100 DVDs. If Geoff already has 27 DVDs, how many more can he purchase that will fit on the shelf?
16. A large cheese pizza costs \$12.50 and each additional topping costs \$1. How many toppings can Max order if he has \$16.00 to spend on a pizza?

Additional Practice *(continued)***Investigation 4****Variables and Patterns**

17. Solve each inequality. Draw a number line and graph each solution.

a. $m \div 5 < 8$

b. $24 - w > 13$

c. $4.5 + x > 5.75$

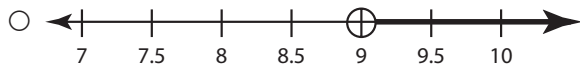
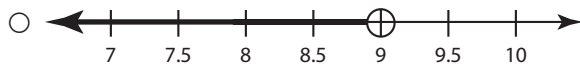
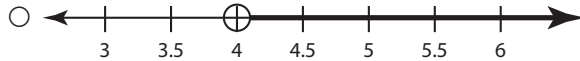
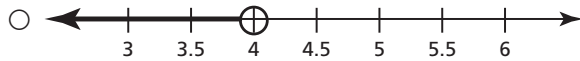
d. $2.5 < 0.5n$

Additional Practice: Digital Assessments

Investigation 4

Variables and Patterns

18. Which of the following number lines represents the solution to $1.5k < 6$?



19. A ski shack offers special pricing on group rentals for a four-hour session and meals. The prices for each option are listed in the table.

Option	Cost
Skis	\$30 per person
Snowboard	\$35 per person
Boots	\$10 per person
Meal ticket	\$7 per person
XL Whole pizza	\$12

Use the tiles to write an expression for each of the following options for a group size of n :

a. Skis

b. Snowboards and an XL pizza

c. Skis, boots, and meal tickets

20. For the inequality $y - 3.5 > 7.1$, determine which values are solutions. Write each value in the box with the correct category.

$y = 11.2$

$y = 3.5$

$y = 10.6$

$y = -2.4$

$y = 2.9$

$y = 12.7$

Solution

Not a Solution

Skill: Finding Solutions**Investigation 4****Variables and Patterns****Solve.**

1. $x + 16 = 31$

2. $4.8 = 2.1 + c$

3. $m + 147 = 207$

4. $29 - k = 12$

5. $p - 4.1 = 2.2$

6. $400 = k - 125$

7. $9a = 72$

8. $3.6 = 0.6x$

9. $10p = 245$

10. $y \div 8 = 11$

11. $7.5 \div g = 1.5$

12. $n \div 30 = 13$

Skill: Finding Solutions *(continued)***Investigation 4****Variables and Patterns**

For each inequality, determine which values are solutions.

13. $k + 2.4 < 5.2$

a. $k = 2$

b. $k = 2.8$

c. $k = 3.1$

14. $8.4 - x > 3.1$

a. $x = 4.5$

b. $x = 5.2$

c. $x = 5.6$

15. $63 < 7p$

a. $p = 7$

b. $p = 8.5$

c. $p = 9.1$

16. $d \div 3 > 5$

a. $d = 14$

b. $d = 18$

c. $d = 20$